Long-term Stability of Chaotic Particles in Hadron Colliders in Simulation and Experiment

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Abstract

In hadron colliders the long-term single particle stability is a crucial issue, in particular for the LHC at injection where the protons will have to survive for some 10 minutes in the presence of strong non-linear fields. The effect of these non-linear fields on the dynamic aperture can be additionally enhanced by tune modulation produced by power supply ripple. The SPS Dynamic Aperture Experiment is performed to test how well simulations can predict the long-term dynamical behavior of particles under these conditions. The dynamic aperture is determined with survival plots in the tracking and found in the experiment using flying wires. A good agreement can be reported. Moreover, in the simulations a resonance could be identified with the surprising property of creating at the same time very chaotic but stable motion. This resonance can attract particles from larger amplitudes due to the tune modulation. Indications for this attraction can also be found in the experimental data.

1 Introduction

At the SPS a systematic experiment has been performed since 1986 to gather experimental data that can be compared with computer simulations and theoretical models. Whereas the first years were devoted to the examination of short-term losses (in the order of seconds) [1] the last years were used to study long-term losses (in the order of minutes) [2].

The experiment is performed in coast to avoid rf noise. The energy is fixed to 120GeV because at this energy the magnets can be operated in a linear regime and space charge effects are negligible. The momentum spread $\Delta p/p$ in the SPS is about 10^{-3} and the normalized emittance (1σ) in both planes $4\mu m$. The beam intensity is kept below 10^{12} protons to avoid collective effects and the closed orbit is well corrected, a rms value of less than 0.3mm in both planes can be achieved. The chromaticity is corrected to $\Delta Q/(\Delta p/p) \approx 1$ and the linear coupling is compensated so that a closest tune approach of $|Q_h - Q_v| = 0.002$ is reached. In the natural tune ripple spectrum there are 7 major lines that add up to a total amplitude of $0.5 \cdot 10^{-4}$, which is half the measured total tune ripple [3]. The machine can be made strongly non-linear by powering 8 sextupoles and a single quadrupole can be used to introduce additional tune modulation that can be made much stronger than the natural one.

We will report results from the working point $(Q_h, Q_v) = (26.637, 26.533)$. After kicking the beam horizontally (by 16.6mm at $\beta_h = 100$m) particles that might be lost are in the tune range that is indicated in Fig. 1 (a). There are two sum resonances of importance: $3Q_h + 4Q_v = 186$ and $8Q_h = 213$. The used ripple frequency is 9Hz (in previous experiments

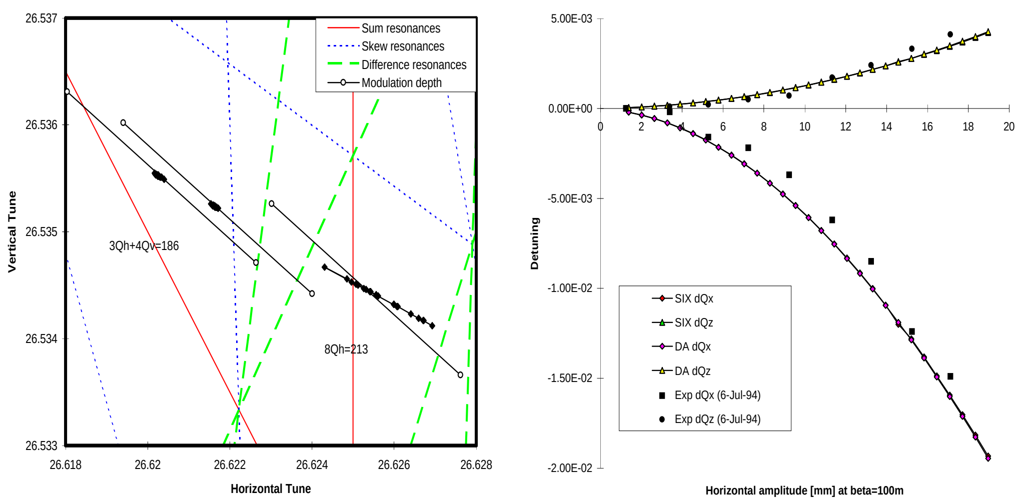
*Supported in part by DESY Hamburg, Germany.

40Hz and 180Hz were tested as well without showing significant differences in the particle loss [4]). The ripple depth used in tracking and experiment is $1.87 \cdot 10^{-3}$ [3]. The amplitude dependent survival time of particles is observed by a flying wire (thickness $8\mu m$) in the experiment. A model of the SPS including all known effects due to magnetic fields is used in the element-by-element tracking simulations [5].

2 Simulations

A first test of the tracking model is provided by the comparison of the amplitude dependent tune obtained from tracking and experimental measurements. Fig. 1 (b) shows that for the working point under consideration, experiment and model are in good agreement.

For the horizontal start amplitude we used 3 values and fixed the vertical amplitude to a probable mean value in all cases. The tracking was done without scraper. As all our starting amplitudes are located in the chaotic region we had to take many particles (640) and concentrate them in a very small domain in phase space.



(a) (b)

Figure 1:
(a) Tune range of interest for the examined working point with resonances up to order 13. The segments show the tune modulation range, the diamonds show the averaged tune per ripple period for the amplitudes 16.8, 18.8 and 19.5mm (right to left).
(b) Horizontal and vertical detuning as function of the horizontal amplitude determined by direct tracking (SIX), normal form analysis (DA) and experiment (Exp).

2.1 Methods

Firstly we applied the Lyapunov method to test the degree of chaoticity. We used a pair of close-by particles (initial phase space distance of 10^{-7}mm) and followed the evolution of their distance in time. In Tab. 1 we give the angular distance between those 2 particles after a given time. Secondly we used the averaged tune per ripple period to relate chaotic behavior to resonances. Thirdly we averaged the amplitudes (again over one ripple period) and studied their mean and rms values as a function of time. Finally we created the standard survival plot using up to 15 million turns.

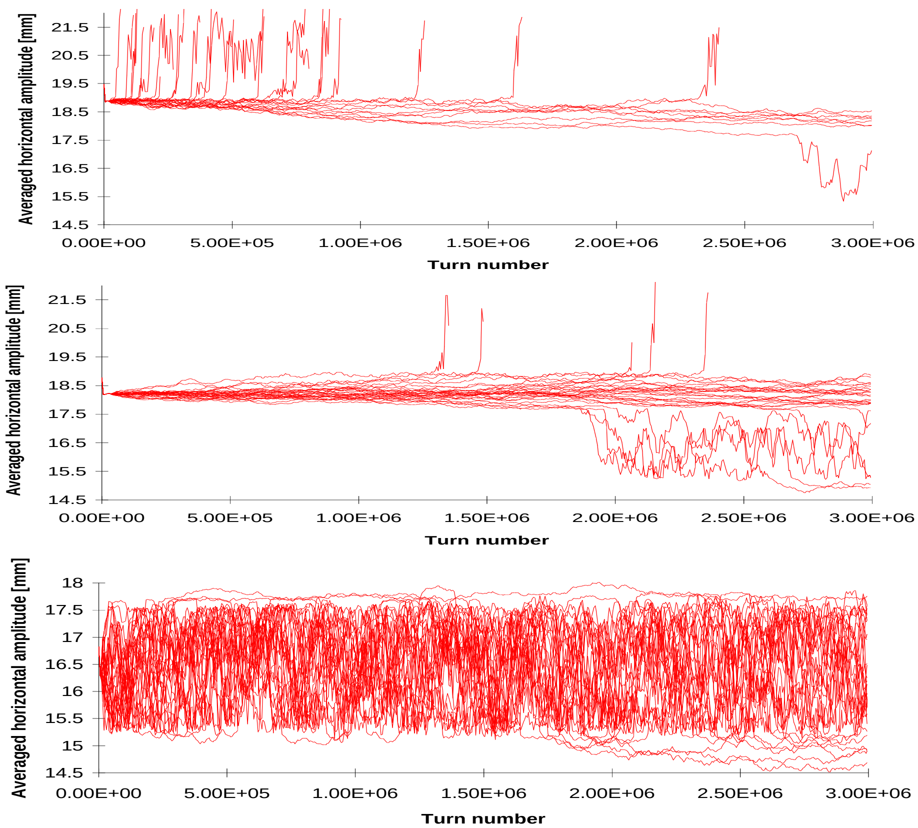


Figure 2: Amplitude evolution of 32 particles starting at A_x =16.8mm, 18.8mm and 19.5mm (bottom to top) under the influence of tune ripple of 9Hz and $\widehat{\Delta Q} = 1.87 \cdot 10^{-3}$. The amplitude is averaged over one ripple period.

2.2 Results

In Fig. 1 (a) all sum, skew and difference resonances up to order 13 are shown in the regime which is relevant for the motion for three selected amplitudes. The averaged tunes per ripple period are depicted together with the modulation depth. At the smallest amplitude (most to the right in Fig.1 (a)) the 8^{th} order resonance is crossed due the tune modulation leading to large fluctuations of the tunes. It is interesting to note that the tunes follow the detuning curve shown in Fig. 1 (b). The largest amplitude (most to the left in Fig.1 (a)) reaches the 7^{th} order resonance.

In Fig. 2 the amplitudes of 32 particles, averaged over one ripple period, are shown for the three different starting amplitude values. At the lowest amplitude (Fig. 2 bottom) the particles fill quickly (less than 10^5 turns) an amplitude band around the 8^{th} order resonance and stay within the band. The size of the band is given by the island size and the ripple depth. Not even one particle out of 640 is lost (Tab. 1). For the second amplitude (18.8mm, Fig. 2 middle) the amplitudes spread slowly (in $10^5 - 10^6$ turns) and finally either reach the 7^{th} order resonance, after which they are extracted in some 10^4 turns, or are attracted down to the 8^{th} order resonance. The particles starting at 19.5mm (Fig. 2 top) fill the same band

Table 1: Particle stability at three different starting amplitudes

horizontal amplitude $[mm]$ at $\beta_h = 100m$	16.8	18.8	19.5
separation of 2 particles $[\pi]$ after 20000 turns,	0.9	0.45×10^{-4}	0.8×10^{-3}
lost particle out of 640, no scraper	0	102	502
amplitude rms value [mm], after 3 million turns, no scraper	0.99	0.72	0.38

as the particles starting at 18.8mm. But since they start closer to the upper band border more particles are lost and less are attracted to the 8^{th} order resonance.

Fig. 3 shows the evolution of the amplitude rms values of the three cases. The very chaotic particles starting at 16.8mm have an immediate increase of the rms value. The deviation from the smooth behavior of the curves in Fig. 3, after 1.7 million and 2.7 million turns for the medium and large amplitude respectively, can be attributed to a decrease of the amplitudes for some particles which get attracted to the 8^{th} order resonance.

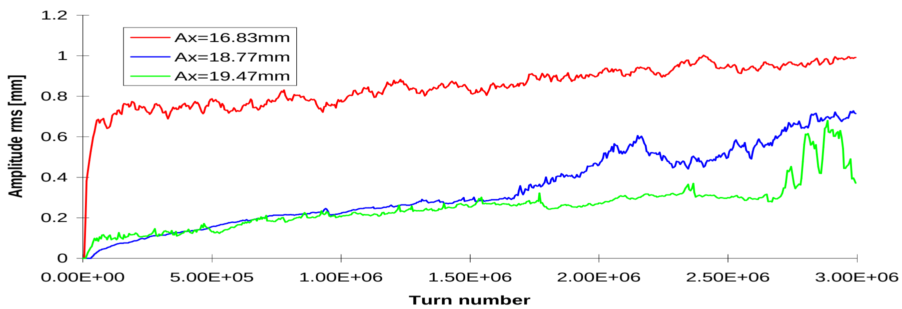
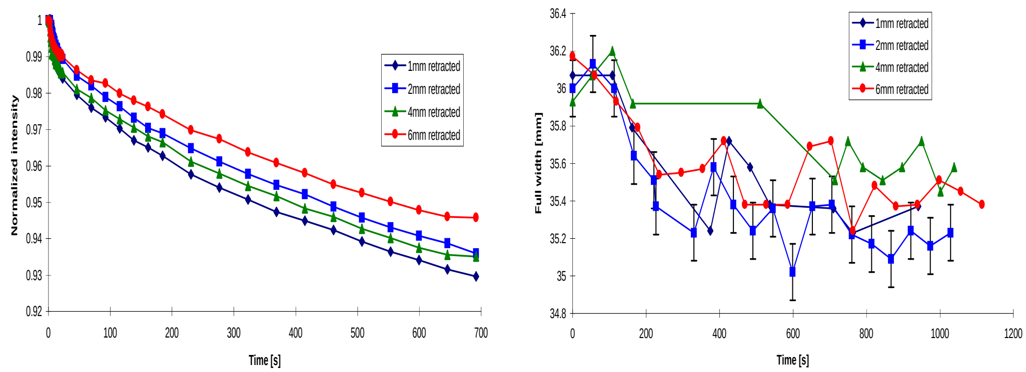


Figure 3: Time dependent amplitude rms values for three different starting amplitudes.

3 Experiments

After a careful adjustment of the machine the beam is kicked horizontally. The vertical and horizontal beam halo is removed by scraping. Then the vertical scraper is retracted completely whereas the horizontal scraper is retracted by 1, 2, 4 or 6mm respectively. The scraper position has always been kept above the upper stability border of the most outside band shown in Fig. 2. Then the tune ripple of 9Hz and $\widehat{\Delta Q_h} = 1.87 \cdot 10^{-3}$ is switched on. In the simulation particles escape in some 10^4 turns when they have crossed the outermost band border. Therefore we measure the probability of passing this border which should be almost independent of the scraper position beyond the outermost band border. We expect that the normalized intensity curves should not differ for those scraper positions.

In Fig. 4 (a) the normalized intensity curves are shown for the different positions of the retracted horizontal scraper. At $t = 0$ the ripple is switched on. After $t = 700$s the difference between the two most extreme curves is only 2% of the start intensity value which confirms our reasoning stated above.



(a) (b)

Figure 4:
(a) Normalized intensity curves for four different scraper positions. At time t=0 the tune ripple of 9Hz and $\widehat{\Delta Q_h} = 1.87 \cdot 10^{-3}$ is switched on.
(b) Time dependent full width of the wire scan signal. At time t=120s a tune ripple of 9Hz and $\widehat{\Delta Q_h} = 1.87 \cdot 10^{-3}$ is switched on.

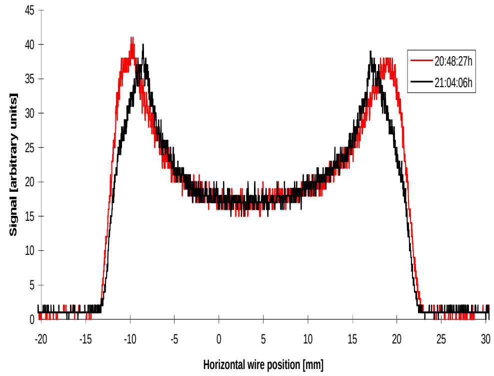
During the experiment a horizontal wire scan was taken about every minute. The full width of this signal is shown in Fig. 4 (b). The ripple is switched on at t = 120s. After a decrease for about 200s the beam size is almost constant for at least 700s (the case at 4mm deviates slightly which may be addressed to some experimental problem).

From the simulations we expect that this regime of stability is caused by two effects: firstly particles are lost when they reach the 7^{th} order resonance at large amplitude and secondly particles at lower amplitudes are attracted towards the 8^{th} order resonance where they survive. The second effect leads to the surprising scenario of an increased particle density at lower amplitude. Indeed, Fig. 5 (a) shows such a scenario: two wire scan profiles are depicted, the first one has been taken just before the ripple is switched on, the second one shows the profile 15.5 minutes later. It is visible that the full width of the signal is slightly reduced but, most remarkably, the signal peaks have moved inwards without loosing height. All four cases with different scraper position have this feature.

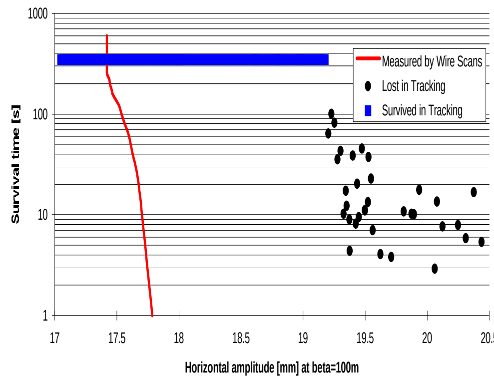
The shrinking of the full width in Fig. 4 (b) can be directly compared with a conventional survival plot of the most stable particles in tracking (on-momentum particles in our case). This is done in Fig. 5 (b) in which particles losses are plotted that have occured before 15 million turns together with the half width from the wire scan profiles. The stability borders from tracking and experiment agree within 10% of the amplitude.

4 Conclusions

Our simulation studies have increased our phenomenological understanding of long-term particle stability. An interesting outcome is the finding that the degree of chaoticity is not a good indicator for the survival time. There is also evidence from simulations and experiment that sizable fractions of particle distributions can move to smaller amplitudes. Of more practical importance is the fact that the dynamic aperture of a machine with known non-linearities and tune modulation can be well predicted by tracking simulations.



(a)



(b)

Figure 5:
(a) Two horizontal wire scans. The first scan was taken before the ripple is switched on, the second scan is 15.5 minutes later.
(b) Survival plot for the examined working point. The vertical line to the left is the experimental stability border taken from time dependent half width of the wire scan signal.

5 Acknowledgment

The authors would like to thank the operations group and all engineers and technicians for the cooperation and help during the experiment. J. Miles has recently joined the experimental team and contributed his expertise in running the SPS. Special thanks go to J. Gareyte for participating in the experiments and inspiring discussions with fresh ideas.

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